## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


CANDIDATE NUMBER

## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 5 Investigation (Core)
May/June 2021
1 hour 10 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 36 .
- The number of marks for each question or part question is shown in brackets [ ].

Answer all the questions.

## INVESTIGATION

## ROLLING SQUARE

This investigation looks at the path of a point on a square as it rolls along the $x$-axis.

A square of side 1 cm rolls along the $x$-axis.
One roll is a turn of $90^{\circ}$ clockwise about its bottom right corner.


Diagram 1 shows the square in Position 1.
One side of the square is bold to help show the rotation.
The centre of the square is $(0.5,0.5)$.
Diagram 2 shows the square rolled $90^{\circ}$ clockwise about $(1,0)$ to Position 2.
1 To get to Position 3 the square rolls $90^{\circ}$ clockwise about (2, 0).
To get to Position 4 the square then rolls $90^{\circ}$ clockwise about $(3,0)$.
(a) On the diagram below, draw the square in Position 4, Position 5 and Position 6.

(b) Complete this table to show the $x$-coordinate of the centre of the square in each position. You may use the diagram to help you.

| Position (n) | 1 | 2 | 3 | 4 | 5 | 6 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$-coordinate | 0.5 | 1.5 | 2.5 |  |  |  |  |

(c) Find the $x$-coordinate of the centre of the square in Position 92.
(d)

(i) The square rolls from Position 1 to Position $n$.

The centre has moved a distance equal to the circumference of 1 circle.
The radius, $r$, of the circle is half the diagonal of the square.
(a) Write down the number of rolls needed.
(b) Write down the value of $n$.
(ii)

| Circumference, $C$, of circle, radius $r$. | $C=2 \pi r$ |
| :--- | :--- |
| Hypotenuse, $r$, of a right-angled triangle <br> with sides $r, x$ and $y$. | $r^{2}=x^{2}+y^{2}$ |

(a) Show that the radius of the circle is 0.707 cm , correct to 3 decimal places.
(b) Find the length of the arc that the centre of the square moves along from Position 1 to Position 2.

2 The side of the square is now 2 cm .


The square rolls along the $x$-axis in the same way as in Question 1.
(a) Complete the table of $x$-coordinates of the centre of the square in different positions.

| Position (n) | 1 | 2 | 3 | 4 | 5 | 6 |  | $n$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$-coordinate | 1 | 3 |  |  |  |  |  |  |

(b) Find the coordinates of the centre of the square in Position 35.
$\qquad$
3 (a) The side of the square is now 3 cm .
Complete the table of $x$-coordinates of the centre of the square in different positions.
You may use the diagram below to help you.

| Position (n) | 1 | 2 | 3 | 4 | 5 | 6 |  | $n$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$-coordinate | 1.5 |  |  |  |  |  |  |  |


(b) The side of the square is now 4 cm .

Complete the table of $x$-coordinates of the centre of the square in different positions.

| Position (n) | 1 | 2 | 3 | 4 | 5 | 6 | $n$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$-coordinate | 2 |  |  |  |  |  |  |

4 Write your expressions from Questions 1(b), 2(a) and 3 in the table below. Complete the table using any patterns you notice.

| Side of square <br> $(w \mathrm{~cm})$ | $x$-coordinate in <br> Position $n$ |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| $w$ |  |

5 A square of side $w \mathrm{~cm}$ rolls from Position 1 to Position 120.
At Position 120, the $x$-coordinate of the centre of the square is 2151 .
Find the value of $w$.

6 A square of side $a \mathrm{~cm}$ is in Position 1.
The coordinates of the centre of the square are $(11, k)$.
(a) Find the value of $k$ and the value of $a$.

$$
\begin{align*}
& k=. \ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{align*}
$$

(b) Find the coordinates of the top right corner of the square.
(...
(c) Write down the $y$-coordinate of the centre of the square in Position 400 .

7 A square rolls along the $x$-axis.

## For the top left corner give a reason why

total distance moved in 2 rolls $=$ total distance moved in 3 rolls.
You may use this grid.


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